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EFFECT OF COMPRESSIVE STRENGTH AND TENSILE REINFORCEMENT RATIO ON FLEXURAL BEHAVIOR OF HIGH-STRENGTH CONCRETE BEAMS-STATIC AND DYNAMIC ANALYSIS

Ali Safaa Ali [1], Mohammad Makki Abbass [2], Yasir W.Abduljaleel[3] and Abdullah Mohammad Makki[4]

[1]Aliraqia University, Iraq, email: alisafaa7873@gmail.com
[2]Ph.D., Consultant Structural Engineering Baghdad, Iraq, email: mohammadmakki2003@gmail.com
[3]Ms.C. , Aliraqia University, Iraq
[4]B.sc, Structural Engineering Baghdad, Iraq, email: abbasmakki89@gmail.com

Abstract

Compressive strength playing an important parameter for all structural members such as beam, slab, column and shear wall. In addition, the reinforcements in the tension zone is important to resist the internal tensile stress that developed due to apply loadings. in present study, the performance of high strength concrete beam under the effect of static loadings is evaluate. Different parameters are considering such as concrete compressive strength of 96.6, 118.3 and 135.5 MPa and different tensile reinforcement ratios of 0.98, 1.47, and 1.97% respectively using finite element approach by ANSYS. Dynamic as harmonic loading is adopting to evaluate the strength capacity and deformation of simply supported reinforced high strength concrete. The models that simulated same as the experiment work as geometry, supports conditions and loadings. Load- deflection, concrete strain, cracks propagations and ductility for all models are discussed. Analysis results indicated that the most important parameter the impact on the performance of high strength concrete beam is rebar’s ratio. The dynamic analysis of such a beams under harmonic loading within 0-50 Hz frequency rage is safe.

Keywords: Compressive Strength, Tensile Reinforcement, Flexural Failure, High Strength Concrete, Dynamic analysis, Harmonic load, Frequency range.

Introduction

Different codes such as American Concrete Institute ACI-318-2019 [1] defined high strength concrete (HSC) as a concrete meeting special combination of performance and uniformity requirements that cannot always be achieved routinely using traditional constituents and normal mixing concrete, placing, and curing practices. HSC have many advantages like reduced cost, time duration of construction and structure would be durable to no corrosion problem In-Hwan Yang et al [2], investigated the behavior of HSC beam under flexural loading. Obtained test results showed that the ductility of such a beam decreased when the tension reinforcement’s ratio increased and the failure mode was crush occur in compressive zone after the tension reinforcement reach yield. Mustafa Kamal Al-Kamal [3], proposed stress block for HSC beam by collected different data base as test results by others. The performance of stress block suggested as triangular rather than rectangular that proposed for normal strength concrete beam. Rashid and Mansur [4], investigated the flexural behavior of high strength reinforced concrete beam with different parameters such as ratio of compressive and tensile strength and concrete compressive strength. Concluded from experimental tests that the crack width and

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the failure load of such concrete adequate when the compressive strength of concrete up to 130 MPa. Jang et al [5], evaluated the ductility of high strength concrete by examined full scale reinforced concrete beam under flexural loadings with different parameters such as reinforcement ratio and concrete compressive strength. Based on studied concluded that the equivalent stress block diagram of concrete cross section lead to reduce the design safety due to increase in concrete compressive strength and reinforcement ratio. Jang et al [6], looked out on the behavior of HSC beam with compressive strength 40 and 70 MPa with different reinforcement ratio that impact on the concrete ductility. Concluded that the presence of stirrups at the bending zone gave more flexural strength and increased in concrete ductility. Hadi1 and Elbasha [7], explored the influence of concrete compressive strength and reinforcement ratio on the HSC beam. Compressive strength in the range of 72-95 MPa with different reinforcement ratio as 5.234-7.86% are considered. Increased in reinforcement ratio and compressive strength lead to increase in ductility due to increase in ultimate deflections so that the ductility index increased. Arslan and Cihanli [8], predicated the curvature ductility of HSC beam based on the compressive strength and reinforcement ratio in addition to reinforcement yield strength. The proposed equation was checkout with several tested beams by others that showed close and find out the ductility increased as the parameters that adopted increased. Maghsoudi and Shari [9], studied the flexural ductility of HSC beam with different amounts of reinforcements ratio in compression and tension zone. Added of reinforcements in compression zone lead to increase in ultimate displacement and rotation. Kwan et al [10], studied the influence of steel confinement on the ductility of HSC beam. Compressive strength and the flexural ductility confinement were adopted as variables. Pointed out that the design of HSC under reinforcement, the confinement not effect on the flexural strength of HSC beam but enhanced the flexural ductility.

Aim and significant of research

The aim of present study is to assessment the flexural capacity, deflections and ductility of high strength concrete simply supported beam with different parameters such as compressive strength and tensile reinforcement ratio under the effects of static and dynamic loadings. The HSC beams subjected to four-point static loading analyzed by analytical method and by using finite elements approach by ANSYS software. Deflection, cracks propagations, strain, ductility and mode of failure for all modeled beams that analyzed and simulated are discussed with details.

Theoretical analysis

The HSC beam design based on the Al-Kamal [3] that adopted in present study. The stirrups distributed each 150 mm along to beam span and there are no stirrups at the middle of the HSC beam. The beam rest on support with distance 150 mm from the beam edge to the center of support of left and right ends. The adopted model lists in Table 1 in which marked and grouped based on the compressive strength and reinforcement ratio. The first six models represent the models without

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steel fiber while B7, B8 and B9 with 1% of steel fiber by volume. Models B10, B11 and B12 same as B1, B5 and B9 but under the impact of harmonic loading.

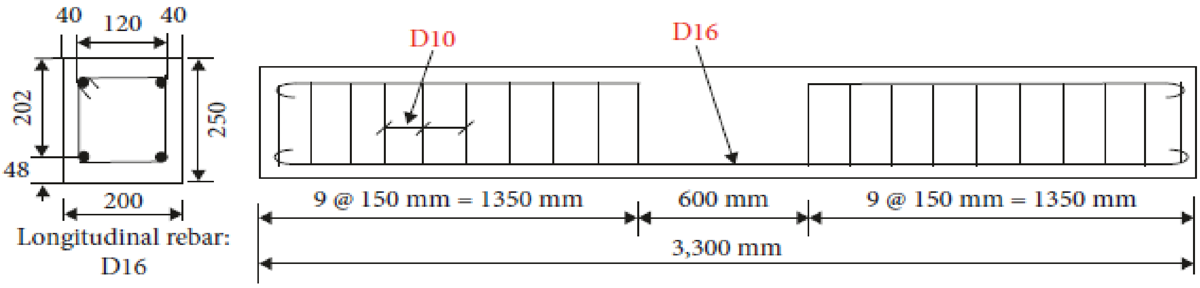

Figure 1: HSC beam details and dimensions [3]

Table 1: Models details

Model mark	Load type	Compressive strength (MPa)	Modulus of elasticity (MPa)	Number of rebar's	% Rebar's ratio ρ	Yielding strength of rebar's (MPa)
B1	Static	96.9	37511	2	0.98	607.9
B2	Static	96.9	37511	3	1.47	607.9
B3	Static	96.9	37511	4	1.97	607.9
B4	Static	118.3	38623	2	0.98	595.8
B5	Static	118.3	38623	3	1.47	595.8
B6	Static	118.3	38623	4	1.97	595.8
B7	Static	135.5	39192	2	0.98	670.0
B8	Static	135.5	39192	3	1.47	670.0
B9	Static	135.5	39192	4	1.97	670.0
B10	Harmonic	96.9	37511	2	0.98	607.9
B11	Harmonic	118.3	38623	3	1.47	595.8
B12	Harmonic	135.5	39192	4	1.97	670.0

The cracking moment calculated by applied equation (1):

$$M_{cr} = \frac{f_r I_g}{y_t} \quad (1)$$

Where $f_r$is the modulus of rupture of concrete; $I_g$is the moment of inertia of the gross concrete section; and $y_t$is the distance of the extreme tension fiber from the neutral axis.The modulus of rupture in case of HSC that suggested by (Rashid, Mansur, and Paramasivam) [11] is adopted to calculate the values of $f_r$as follow:

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$$f_r = 0.42(f_c^{'})^{0.68} \quad 2$$

The maximum service deflection at mid-span by applied equation (3):

$$\delta_s = \frac{M}{24E_c I}(3L^2 - 4a^2) \quad 3$$

The stiffness of concrete beam becomes less after the initiation of the first crack due to the contribution of the concrete to the stiffness of the beam is reduced in the cracked section. The crack stiffness of the beams models calculated as follow [12]:

$$k_{cr} = \frac{P_y - P_{cr}}{\delta_y - \delta_{cr}} \quad 4$$

The ductility index of the beam models was calculated that suggested by Singh et al. [13] as follow:

$$\mu = \frac{\delta_p}{\delta_y} \quad 5$$

Finite element modeling

The actual load capacities were taken from the previous test [3] that adopted and applied to the models that is simulated by ANSYS [14]. The loads were applied under four points and the models were run as static analysis for nine models and three extra models under the effect of harmonic loading with frequency ranged 0-50 Hz. The model is divided into a numbers of small elements, 44 elements longitudinal direction (each element is 75 mm), 10 elements in width and 5 elements in depth directions that mean each element is 50 mm. All lines within the beam model are divided to produce meshes, lines mesh adopted after many trails to select the mesh size to get near close solutions. The connection between rebar nodes is similar to the concrete solid nodes, so that the concrete and steel reinforcement nodes are merged (full interaction, no slip and friction). The tolerance value of 0.05 is used as displacement control during the nonlinear solution for convergence. Numerical analysis using finite elements method by ANSYS software version 20 was used to simulate all HSC beams with and without presences of steel fibers. Different elements were selected to simulate the real performance of concrete, support conditions, plates under applied loadings, stirrups and rebar’s. SOLID65 element used for concrete material in which three degrees of freedom at each nodes plus translations. LINK180 element is adopted to simulate all steel reinforcement. SOLID185 is chosen to represent the steel plates that locates under the applied loads and supports [17]. Smeared crack is the best representation of reinforced concrete members such as adapt beam. The open and close coefficients for concrete cracks were 0.2 and 0.7 respectively. The materials nonlinearity for steel rebar’s and concrete are behaved as elastic – full plastic reinforcements, concrete

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linear up to 0.3fc’, elastic up to 0.85 fc’, maximum value of concrete strain is 0.003. The main assumptions of numerical analysis for the plane section remain plane before and after applied loads, the concrete is homogeneous, full bounds between concrete and reinforcements and the self-weight of beam not considered in analysis that match the experimental tests. Figure 2ashows the three dimensional HSC beam model, Figure 2bshows the meshes and Figure2c shows the wireframe that represent main and stirrup rebar’s. The loads are divided based on the nodes at the top of steel plates that distributed as point loads. The supports conditions of all HSC beams are simply supported, the left support is roller and the right support is pin. The loads were applied at the central upper nodes that located at the tops of steel plates in which the loads with magnitude same as experimental tests.

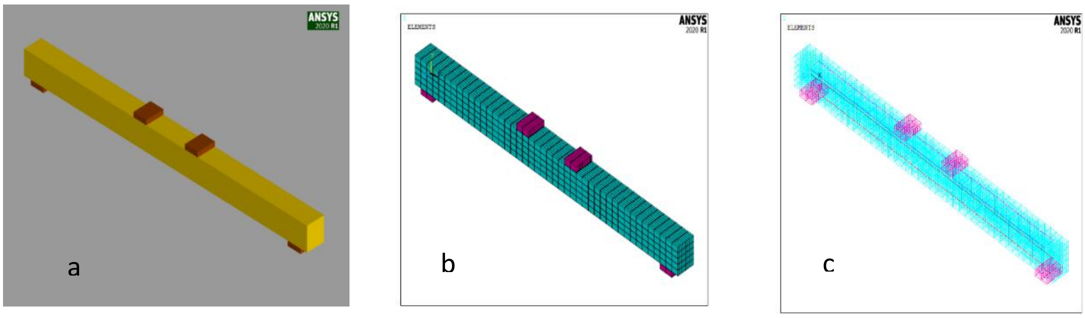

Figure 2: (a) Three dimensional HSC model, (b) Model meshes and (c) wireframe model

shows main and stirrup rebar’s

Analysis results

The static analysis results of HSC models that listed in Table 2 to 4. Analysis results as deflection, longitudinal strain at concrete and cracks propagations at yielding load stage are presented through Figures 3 to 5 shows the applied load with deflection variations for the models B1 to B9. A Figure 6 to 14 shows the deflection, strain at concrete and cracks propagations at yielding stage for models B1 to B9. The comparisons list in Tables 2 to 4 shows that the mean value rounded to unity and the standard deviation so small that is mean all results rounded to average value in addition to the variance is low so that the point of numerical and experimental tests so close.

A Figure 3 to 5 shows the performance of HSC beam under the effect of static loading and the corresponding deflections and an additions of deflections at first crack loading and the service deflection of simply supported beam that equal to span/360. The models behave at first crack loading as linear and after the inflection point become nonlinear. Increase of load lead to increase in deflection so that the stiffness of the beam becomes less and the curve become toward the horsetail axis.

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The comparisons between the models there are increase in load capacity in cases of increase in compressive strength and rebar's ratio. Figures 6 to 14 shows the deflections, strains in concrete and cracks propagations in three dimensions of HSC beam at yielding stage.

Table 2: Comparisons between experimental and numerical analysis results as critical load, crack moment and deflection at first crack load

Model mark	Initial cracking state [3]			Initial cracking state ANSYS			%Ratio (ANSYS/Experimental [3]		
	P_{cr} (kN)	M_{cr} (kN.m)	Δ_{cr} (mm)	P_{cr} (kN)	M_{cr} (kN.m)	Δ_{cr} (mm)	P_{cr}	M_{cr}	Δ_{cr}
B1	16.1	9.6	0.9	17.94	10.28	0.97	1.11	1.07	1.08
B2	16.4	9.8	1.0	18.10	10.68	1.02	1.10	1.09	1.02
B3	12.7	7.6	0.9	12.96	7.65	0.95	1.02	1.01	1.06
B4	23.3	14.0	1.2	24.12	14.23	1.31	1.04	1.02	1.09
B5	17.4	10.4	0.9	17.85	10.53	0.94	1.03	1.01	1.04
B6	17.4	10.4	1.0	20.92	12.34	1.15	1.20	1.19	1.15
B7	46.4	27.8	2.9	38.25	22.57	3.22	0.82	0.81	1.11
B8	62.2	37.3	5.1	52.5	31.00	5.59	0.84	0.83	1.10
B9	58.8	35.3	4.2	49.75	29.36	5.20	0.85	0.83	1.24
Mean							1.00	0.98	1.09
Standard deviation							0.14	0.13	0.06
Variance							0.02	0.02	0.01

Table 3: Comparisons between experimental and numerical analysis results as yielding load, yielding moment and deflection at yielding load

Model mark	Yielding state [3]			Yielding state ANSYS			%Ratio (ANSYS/Experimental [3]		
	P_y (kN)	M_y (kN.m)	Δ_y (mm)	P_y (kN)	M_y (kN.m)	Δ_y (mm)	P_y	M_y	Δ_y
B1	78.0	46.8	27.3	78.0	46.8	22.11	1.00	1.00	81
B2	129.0	30.5	30.5	129.0	30.5	24.56	1.00	1.00	81
B3	162.0	32.6	32.6	162.0	32.6	25.11	1.00	1.00	77
B4	86.4	51.8	22.2	86.4	51.8	23.95	1.00	1.00	107
B5	120.4	72.2	26.0	120.4	72.2	22.95	1.00	1.00	88

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B6	157.2	94.3	28.5	157.2	94.3	24.14	1.00	1.00	85
B7	145.1	87.1	22.3	145.1	87.1	24.68	1.00	1.00	110
B8	172.4	103.4	22.8	172.4	103.4	25.38	1.00	1.00	111
B9	222.0	133.2	25.7	222.0	133.2	28.13	1.00	1.00	109
Mean							1.00	1.00	0.95
Standard deviation							0.00	0.00	0.148
Variance							0.00	0.00	0.022

Table 4: Comparisons between experimental and numerical analysis results as crack stiffness load and ductility index

Model mark	Experimental [3]		ANSYS		%Ratio (ANSYS/Experimental)	
	Crack stiffness k_{cr}(kN/mm)	Ductility index	Crack stiffness k_{cr} (kN/mm)	Ductility index	Crack stiffness k_{cr}	Ductility index
B1	2.35	3.38	2.84	4.17	1.21	1.23
B2	3.82	1.47	4.71	1.83	1.23	1.25
B3	4.71	1.47	6.17	1.90	1.31	1.29
B4	3.00	4.34	2.75	4.01	0.92	0.92
B5	4.11	2.35	4.66	2.65	1.13	1.13
B6	5.09	2.26	5.93	2.67	1.16	1.18
B7	5.09	1.37	4.98	1.24	0.98	0.90
B8	6.23	1.43	6.06	1.28	0.97	0.90
B9	7.59	1.60	7.51	1.46	0.99	0.91
Mean					1.10	1.08
Standard deviation					0.14	0.17
Variance					0.02	0.03

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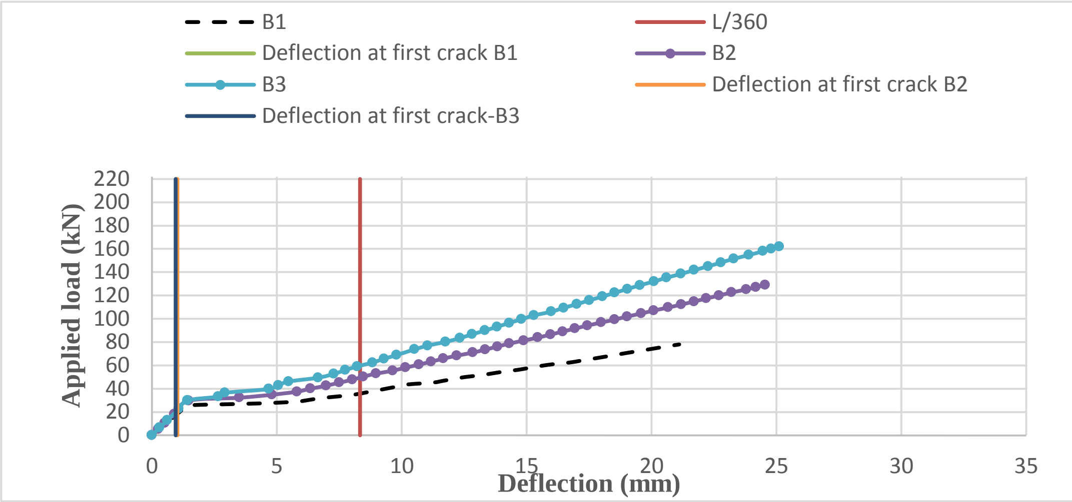

Figure 3: Load deflections of models B1, B2 and B3

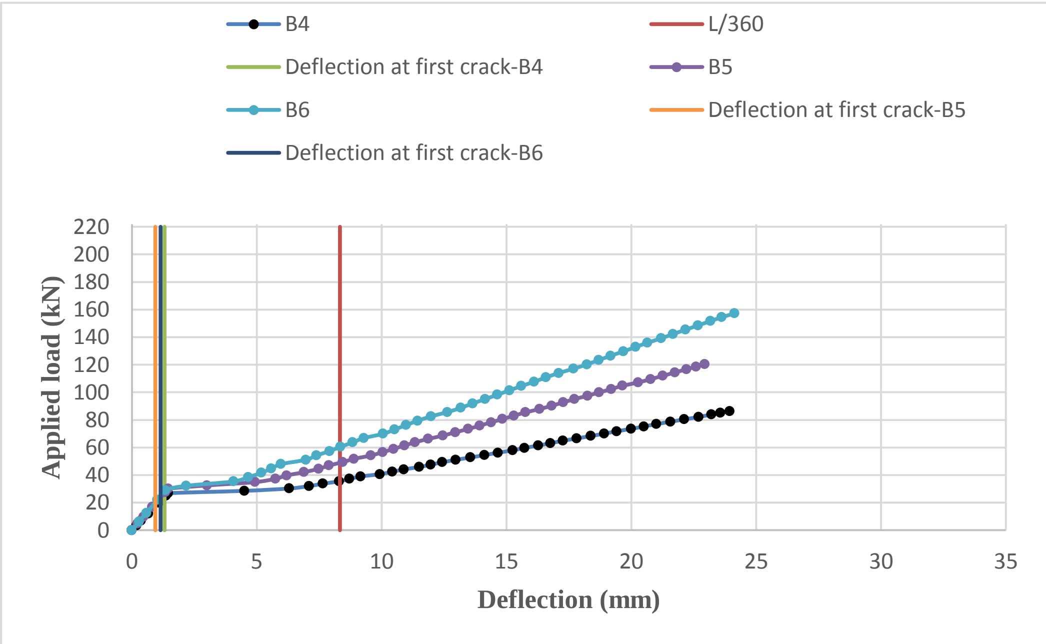

Figure 4: Load-deflections of models B4, B5 and B6

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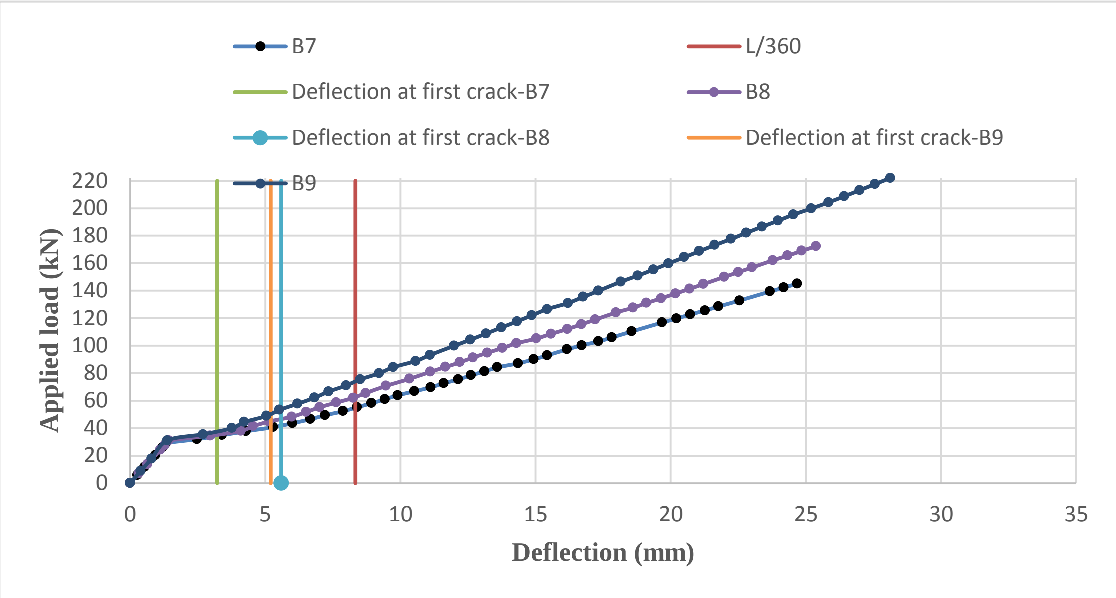
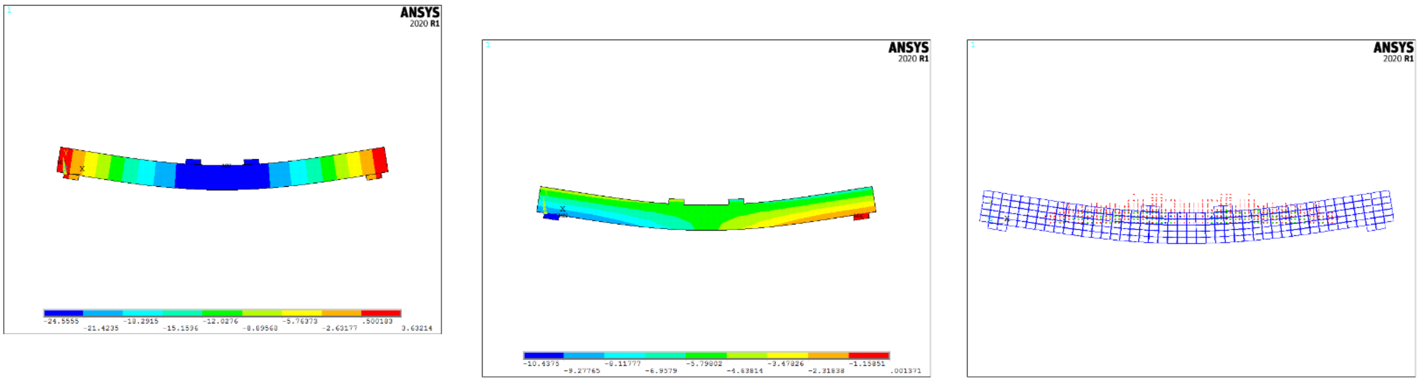


Figure 5: Load-deflections of models B7, B8 and B9

Figure 6: Deflection, Displacement along beam and cracks propagations-B1



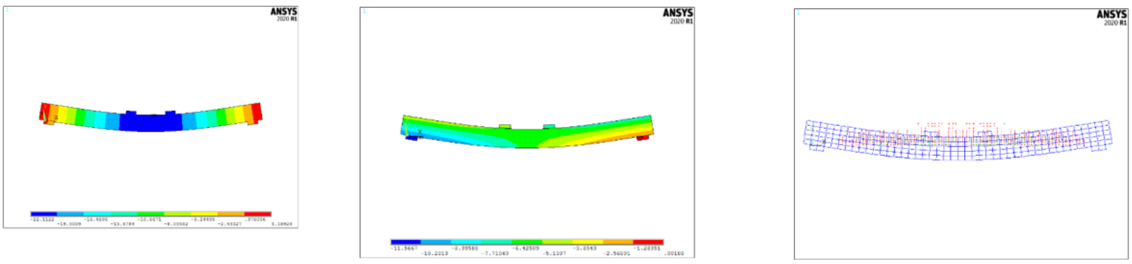


Figure 7: Deflection, Displacement along beam and cracks propagations-B2

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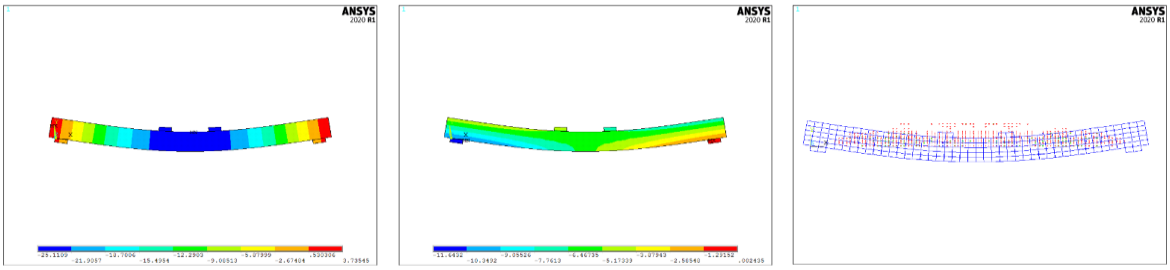

Figure 8: Deflection, Displacement along beam and cracks propagations-B3

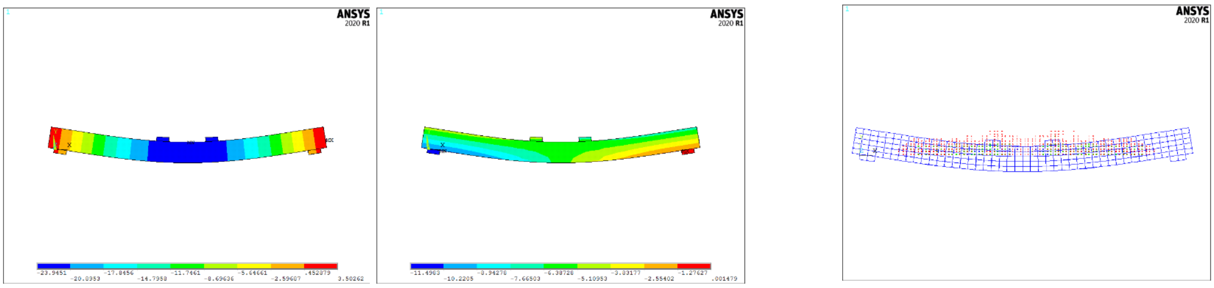

Figure 9: Deflection, Displacement along beam and cracks propagations-B4

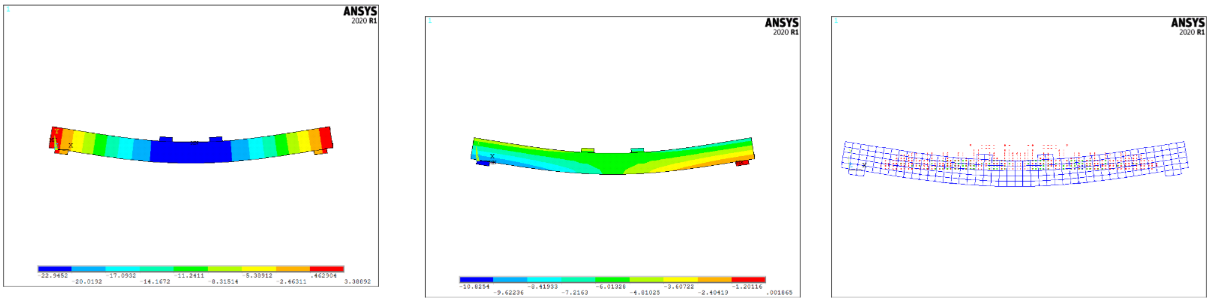

Figure 10: Deflection, Displacement along beam and cracks propagations-B5

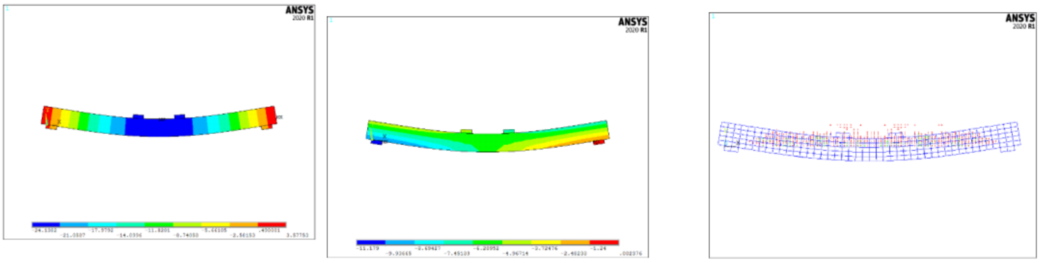

Figure 11: : Deflection, Displacement along beam and cracks propagations-B6

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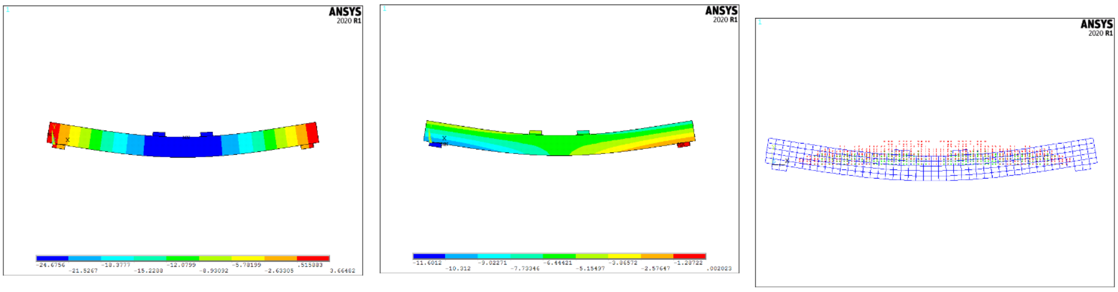

Figure 12: Deflection, Displacement along beam and cracks propagations-B7

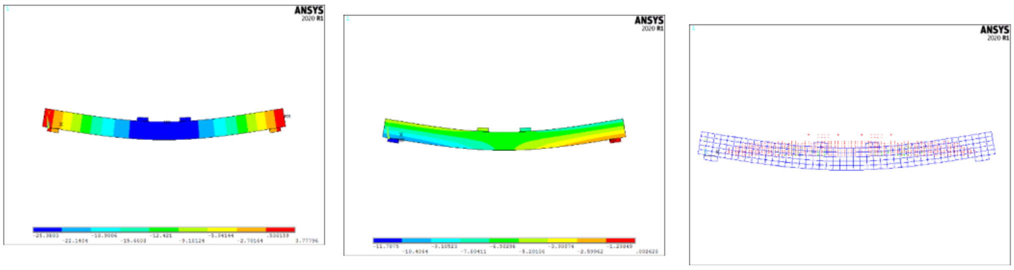

Figure 13: Deflection, Displacement along beam and cracks propagations-B8

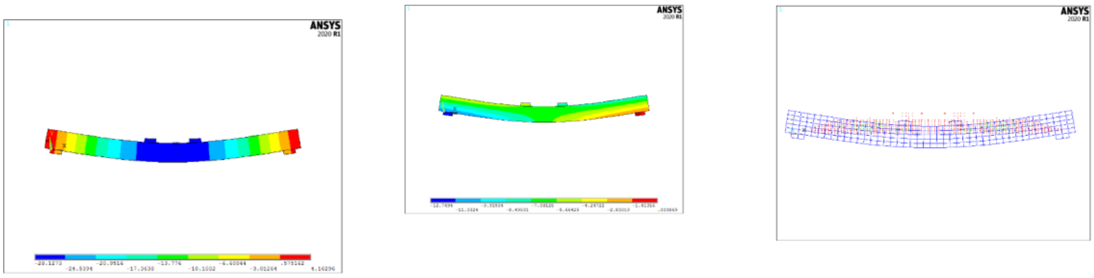

Figure 14: Deflection, Displacement along beam and cracks propagations-B9

Dynamic analysis - Harmonic – frequency range

Harmonic analysis is applied to check out the performance of GCS beams under the effect of this type of dynamic loading with frequency range 0-50 Hz with 1 Hz as frequency step. This loading represents a wave that adopted as generalization of the notions of Fourier series and Fourier transforms. Same applied load as static loadings were applied and read the deflection at the mid-span of the HSC beam at the same location that read it in static analysis to evaluate the performance of HSC

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beam under the impact of harmonic loading. Table 5 lists the frequency adopted range, absolute maximum deflections and at which frequency that occur. Figures 15 to 17 shows the variations of deflection with frequency range of beams B10, B11 and B12 in which Figure 18 represent the performance of all HSC beams (B10, B11 and B12).

Table 5: Frequency range and maximum deflection of models B10, B11 and

Model mark	Frequency range *f* (Hz)	Maximum deflection (mm)	Frequency at maximum deflection (Hz)
B10	0-50	8.25	1.00
B11	0-50	10.35	1.00
B12	0-50	17.51	1.00

B12

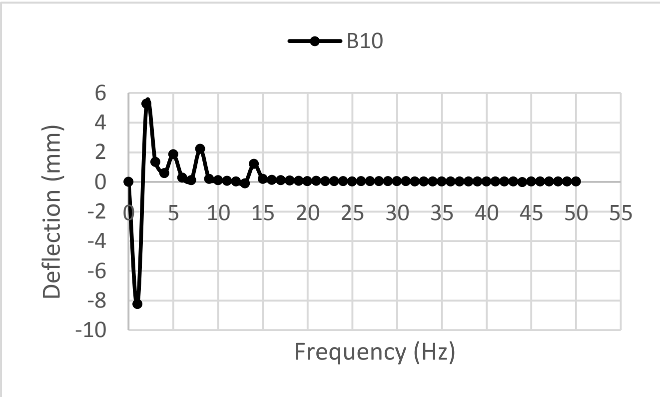

Figure 15: Deflection-frequency variations at mid-span of HSC model B10

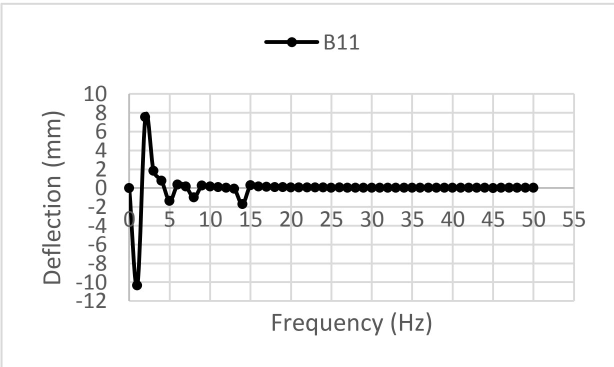

Figure 16: Deflection, Displacement along beam and cracks propagations-B2

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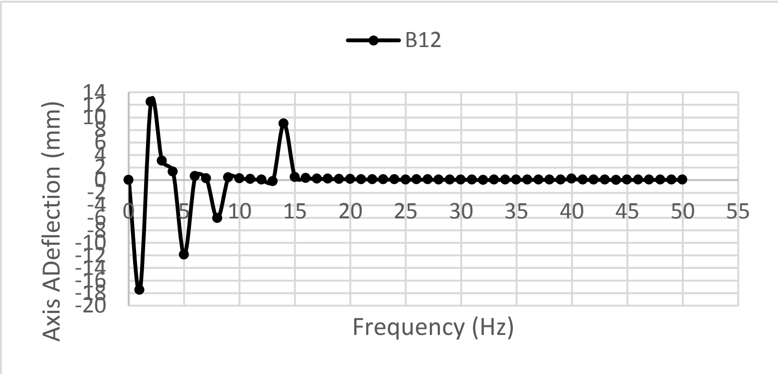

Figure 17: Deflection-frequency variations at mid-span of HSC model B12

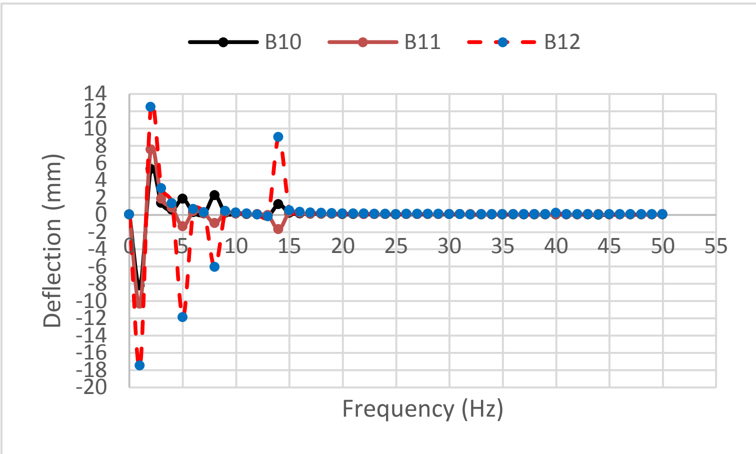

Figure 18: Deflection-frequency variations at mid-span for all HSC models

The magnitude of deflection at the center of the HSC beam model depending on the value of frequency in which at each frequency there is certain deflection. The maximum central deflection for all models occur at 1 Hz frequency that differ in magnitude for the three models relies on the compressive strength and rebar's ratio. When the increase in compressive strength and rebar's ratio that lead to decrease in central deflection.

Discussions and conclusions

Based on the finite element analysis results that adopted ANSYS software to simulate the experimental HSC beams by other researchers, following are the most important points as discussions and conclusions that summarized from present study:

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Effect of compressive strength

The effect of compressive strength on the strength capacity and deflection at yielding stage summarized in Table 6 in which each control model load projected on the other model load-deflection to find out the value of deflection for the control model load. The deflection decreased as the compressive strength increased due to the model become more resistance in tension zone and reduce cracks due to increase in modulus if rupture because of it is function of compressive strength. Increase in compressive strength lead to increase in modulus of elasticity so that the deflection becomes less. The presences of steel fiber within the concrete matrix make the concrete more strength to resist the applied loads due to it is working like bridge that connecting the concrete particles and to make the concrete more ductile so that improvement the strength capacity and enhance the concrete to resist tensile strength that develop in tension zone so that reduce cracks sol that the cracks propagations in case of higher compressive strength are less in intensity.

Table 6: Effect of compressive strength on the strength and deflection of models

Model mark	Yielding state ANSYS		% (+) increase and (-) decrease	
	P_y (kN)	Δ_y (mm)	(+) P_y	(-) Δ_y
B1	78.0	22.11	---	---
B4	86.4	21.00	10.77	4.89
B7	145.1	12.6	86.03	42.99

Effect of rebar's ratio

The effect of rebar's ratio on the strength capacity and deflection at yielding stage lists Table 7 in which each control model load for each group compared with other models as reference. The deflection decreased as the rebar's ratio increase due to make the concrete more ductile and gave more resistance to the applied load so that the strength capacity increase. The cracks stiffness increase when the rebar's ratio increase due to increase in first crack load because of the model become more ductile.

Table 7: Effect of rebar's ratio on the strength and deflection of models

Model mark	Yielding state ANSYS		% (+) increase and (-) decrease	
	P_y (kN)	Δ_y (mm)	(+) P_y	(-) Δ_y
B1	78.0	22.11	---	---
B2	129.0	14.29	65.38	35.37

B3	162.0	11.50	107.69	47.99
B4	86.4	23.95	---	---
B5	120.4	15.71	39.35	34.41
B6	157.2	12.65	81.94	47.18
B7	145.1	24.68	---	---
B8	172.4	21.20	18.81	14.1
B9	222.0	18.15	53.00	26.46

Effect of loading type

According to the analysis results for selected models that lists in Table 8, the beam models gave deflection at the center of the span of HSC beam under harmonic loading less than the applied static loadings so that these models can sustain and carrying the applied dynamic loadings.

Table 8: Effect of loading types on the deflection of models

Model mark	**Load type**	**Maximum deflection (mm)**
B1	Static	22.11
B5	Static	22.95
B9	Static	28.13
B10	Harmonic	8.25
B11	Harmonic	10.35
B12	Harmonic	17.51

Based on the analysis results, the most affects parameter on the behavior, strength and deformation such as deflection, strain and cracks propagations is the rebar's ratio. the models that simulated to check out the deformation such as deflection under dynamic loading gave deflection less than that occur at yielding stage but more than span/360 so that it safe bur require some treatments such as rubbers under supports and under applied loads or increase the beam stiffness or providing damping to prevent resonance and reducing the deflection.

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